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CMSE114312023

AND

PRESCRIPTIVE ANALYTICS WITH MATHEMATICAL PROGRAMMING

INDIVIDUAL ASSIGNMENT

B244512

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## Q1:

(A):

**Parameters:**

1. and represent distribution centres (DCs) and demand zones, respectively.
2. and represent the number of distribution centres and demand zones, respectively.
3. : Transportation costs from the distribution centerto demand zone .
4. : Distribution capacity at distribution center.
5. : Demand at demand zone .
6. : Penalty for unmet demand, set at £500 per unit.
7. : Shipping cost, set at £3 per mile.

**Decision Variables:**

: Quantity of goods to be shipped from distribution centre to demand zone .

: Unmet demand at demand zone .

**Objective Function:**

(B):

After implementing the above model in GAMS and solving it using MIP, I came up with the following analysis based on the optimal solution:

1. As for the **objective function**, The optimal solution results in a total cost of £281,740.
2. As for the **decision variables**,

Quantities Shipped ():

Gourock: 150 units to Perth

Bathgate: 150 units to Perth

Dunfermline: 100 units to Perth

Edinburgh: 200 units to Perth

Portlethen: 400 units to Dundee

Unmet Demand ():

Dundee: There is an unmet demand of 200 units.

1. As for **service level**, The demand in Perth (600 units) is fully met by the distribution centres. The demand in Dundee is partially met (400 units supplied), resulting in an unmet demand of 200 units. This implies a service level issue for Dundee, as about 33.3% of its demand is not satisfied.

(C):

**shadow prices:**

As for the capacity constraint, we can learn that: If we want to reduce the total cost, we can do it by increasing the capacities of the distribution centres. For example, if we increase the capacity of Gourock by 1 unit, the total cost of the optimal solution will be reduced by £158.

As for the demand constraint, we can learn that: increasing the demand of distribution centres will result in the increasing of total cost. For example, if we increase the demand of Perth by 1 unit, the total cost of the optimal solution will be increased by £476.

**reduced costs:**

As for the quantity of goods to be shipped from the distribution center to demand zone (), we can learn that: starting to ship a unit of goods along that route (which is currently not used) would increase the total cost by the amount of the reduced cost. For example, if you start shipping from Bathgate to Dundee, the total cost would increase by £12. This indicates that under the current optimal solution, it's not cost-effective to initiate shipping along these routes.

As for unmet demand at demand zone (), we can learn that: unmet demand in Perth is £24. This means if the unmet demand in Perth increases by one unit, the total cost would increase by £24.

(D):

**Objective function:**

With zero capacity at all distribution centres, the total cost will be entirely due to penalties for unmet demand. Since the total demand of demand zones is 1200 units and the penalty for one unit is £500, the total cost would be £600,000.

**Decision Variables:**

As for the quantity of goods to be shipped from distribution centre to demand zone (), they would be all zero. As for unmet demand at demand zone (), both Perth and Dundee would be 600 units.

(E):

Let and represent the unmet demands in Dundee and Perth, respectively. The new constraint can be expressed as:

The constraint aims to balance the service level between the two demand zones, ensuring that one area's unmet demand does not significantly exceed the others. It helps prevent excessive neglect of one demand zone over the other.

As for its implications on the optimal solution, it could impact the total cost. Meeting this constraint might require increasing the supply to one area, leading to increased total costs.

(F):

I would like to include some algorithm techniques for this question.

1. Greedy Algorithm. When solving optimization problems that are not overly complex and have a relatively straightforward solution approach, we can use a greedy algorithm. This is applicable in cases where the overall problem can be broken down into subproblems, and local optimal solutions lead to a global optimal solution.
2. DFS and BFS. When using the branch-and-bound method to solve optimization problems, DFS can help us by deeply exploring along a path to quickly find an optimal solution. Similarly, BFS is suitable for scenarios where the optimal solution is in the shallower layers of the search space.

## Q2:

(A):

Firstly, if Capt = 0, then Xij for all time period will all be 0. Additionally, the 4th constraint makes sure that there is no backlog at the end of the last time period (when t = T).

1. **With the 4th constraint:** Given constraint (2), for all time period before the last period T, we can pass the unmet demand using backlogging to the next time period. For example, let’s assume we have 3 time period with demand =10, 20, 30, and only 1 product whose index = 1. In the 1st time period, Since=0, =0. Initial inventory =0, no backlog =0. Unmet demand =10 becomes backlog =10. New inventory =0 (because of no production). So the constraint (2) would be: . In this way, we can keep passing the unmet demand to the last phase as backlogs, but due to 4th constraint these backlogs turn into lost sales.
2. **Without the** **4th constraint:** In the first case, the demand in the last period becomes lost sales, while in the second case, all unmet demands accumulate as backlog because of the removal of the4th constraint.

(B):

(i):

(ii):

(iii):

(C):

**Linear relaxation:**

**Relationship:** After coming up with the optimal solution with linear relaxation, we get the starting point for branch-and-bound algorithm, for example, in our linear relaxation optimal solution, . And then we can select one variable to branch on to test out rounding possibilities, for example, for we can set its value to or as our branches. We repeat this process of linear relaxation and solving. The solution of each branch gives a lower bound. And if we get a better new solution, we update the current best solution, otherwise we prune this branch. We keep doing this until we find the feasible optimal integer solution for our problem.

(D):

The root node of our tree is the optimal solution with linear relaxation, for example, . And then we test out ’s value between 0 and 1 in the following branches. For , one branch would set =0, and another branch would set =1. Similarly, for , branches would be created setting =0 and =1, respectively. Each branch forms a new subproblem, and these subproblems are then subject to linear relaxation and solving again, progressively moving towards an integer solution.

(E):

To evaluate, I would like to use the primal gap to measure the distance between a feasible (integer)

solution and the optimal solution.

The smaller the gap, the closer our current integer solution is to the optimum.

## Q3:

(A):

**Indexes:** we have 3 indexes: : Types of crops, : States of India, : Agricultural periods

**Parameters:**

: Yield of crop in state during time , : Price of crop , : Demand for crop , : Land area of state during time , : Seed cost for crop , : Seeding rate for crop , : Average daily wage in state , : Working days during time , : Number of workers needed in state , : Fertilizer cost for crop, : Fixed cost for machine renting in state during time

**Decision Variables:**

: Land area allocated for crop in state during time , : Binary variable indicating if any crop is planted in state during time

**Objective function:**

Maximizes total profit, calculated as the sum of income from crops minus the costs of planting, including labour, seed, fertilizer, and fixed costs.

**Constraints:**

We defined the following constraints related to crop demand, land use rates, and planting area, etc.

(B):

**Model limitations:**

1. The current model doesn’t take climate uncertainties into account, for example, natural disasters like floods or droughts.

2. The current model assumes the prices of crops are fixed, which is not reasonable and neglects market fluctuations.

**Extended version:**

To tackle the limitations I mentioned above, I intend to make the following modifications:

1. Introduce to represent the climate impact factor for state in period .
2. Replace the original with to denote the price of crop in period , accounting for market dynamics.

So, the extended version of the objective function would be like this:

**Benefits to stakeholders:**

The updates will improve the risk management ability of the model, incorporating crop prices’ fluctuations and climate factors help farmers better manage risks and make more robust decisions to tackle upcoming natural disasters and prevent potential financial loss.